

2.

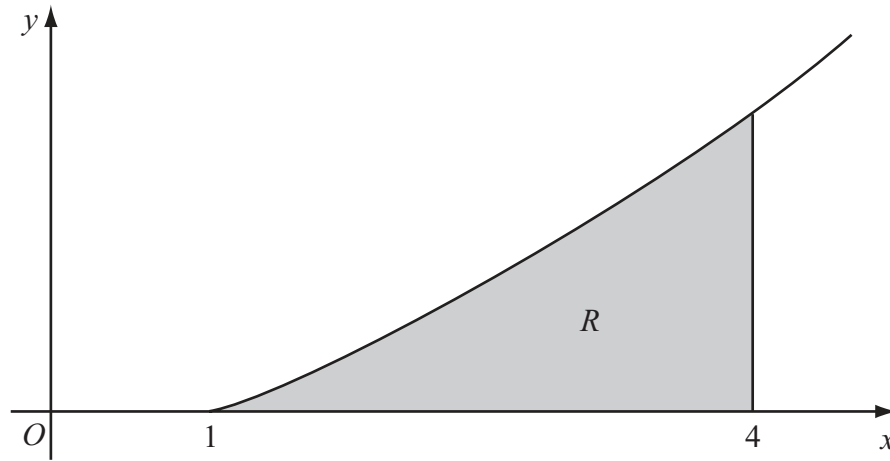


Figure 1

Figure 1 shows a sketch of the curve with equation $y = x \ln x$, $x \geq 1$. The finite region R , shown shaded in Figure 1, is bounded by the curve, the x -axis and the line $x = 4$.

The table shows corresponding values of x and y for $y = x \ln x$.

x	1	1.5	2	2.5	3	3.5	4
y	0	0.608			3.296	4.385	5.545

- (a) Complete the table with the values of y corresponding to $x = 2$ and $x = 2.5$, giving your answers to 3 decimal places. (2)
- (b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R , giving your answer to 2 decimal places. (4)
- (c) (i) Use integration by parts to find $\int x \ln x \, dx$.
- (ii) Hence find the exact area of R , giving your answer in the form $\frac{1}{4}(a \ln 2 + b)$, where a and b are integers. (7)



7.

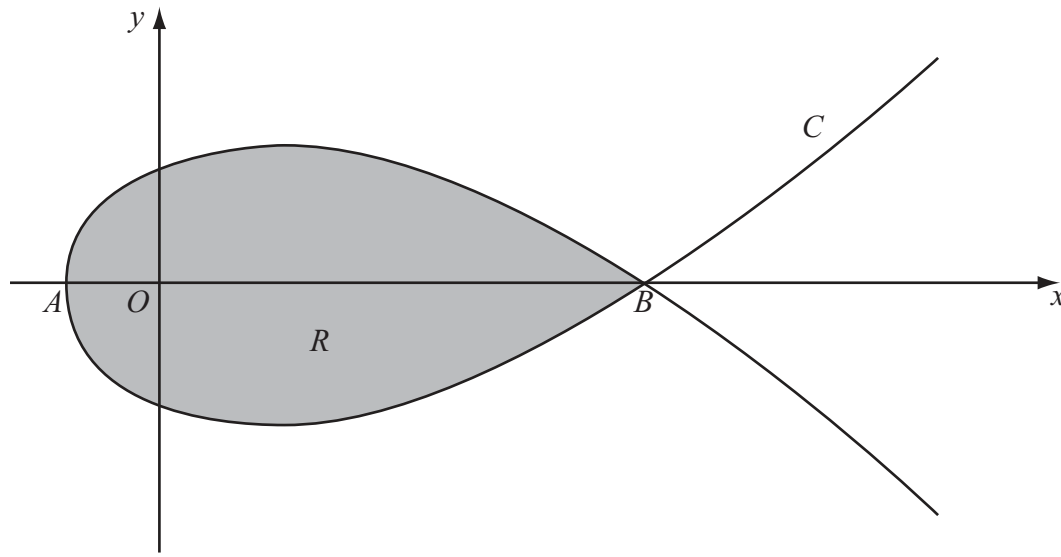


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \quad y = t(9 - t^2)$$

The curve C cuts the x -axis at the points A and B .

- (a) Find the x -coordinate at the point A and the x -coordinate at the point B . (3)

The region R , as shown shaded in Figure 2, is enclosed by the loop of the curve.

- (b) Use integration to find the area of R . (6)



8. (a) Using the substitution $x = 2 \cos u$, or otherwise, find the exact value of

$$\int_1^{\sqrt{2}} \frac{1}{x^2 \sqrt{4-x^2}} dx \quad (7)$$

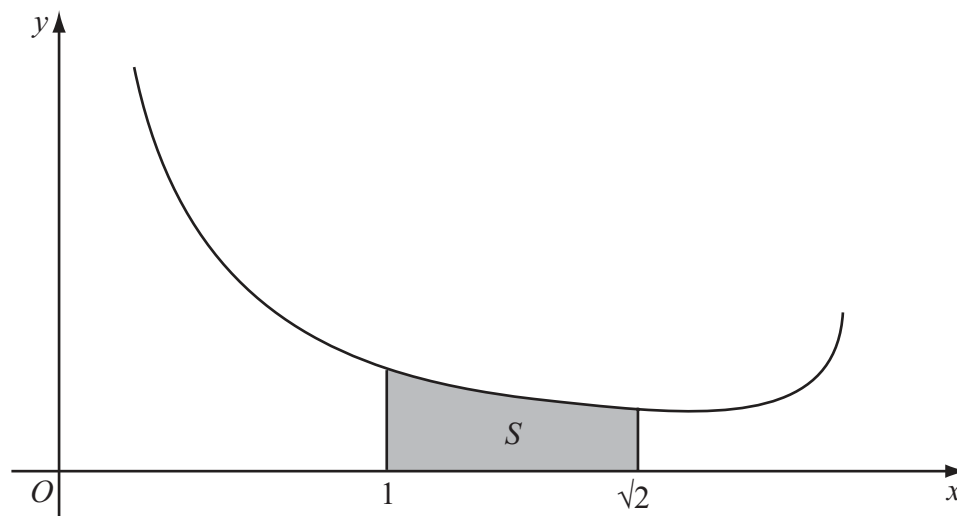


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = \frac{4}{x(4-x^2)^{\frac{1}{2}}}$, $0 < x < 2$.

The shaded region S , shown in Figure 3, is bounded by the curve, the x -axis and the lines with equations $x = 1$ and $x = \sqrt{2}$. The shaded region S is rotated through 2π radians about the x -axis to form a solid of revolution.

(b) Using your answer to part (a), find the exact volume of the solid of revolution formed. (3)



